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HEAT TRANSFER IN A LAYER OF LIQUID ON A ROTATING ARCHIMEDES SPIRAL TAKING ACCOUNT OF THE ENTRANCE REGION

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UDC 532.526.75

The effect of the entrance region on the hydrodynamics and heat transfer in a layer of liquid on a rotating surface is studied.

Hydrodynamics and mass transfer in a layer of liquid on a rotating Archimedes spiral in the absence of wave formation were investigated earlier [1] by the integral relations method. In the present article we use the work method [2] to study heat transfer in a laminar liquid film on an Archimedes spiral rotating with a constant angular velocity ω, taking account of the entrance region.

We choose the origin of coordinates in the plane of the outlet, the x axis along the flow, and the y axis normal to it. The x, y coordinate system is fixed with respect to the streamlined solid surface. It is assumed that the pressure gradient in the liquid layer is produced by the rotation of the spiral apparatus and that the longitudinal rate of change of the flow parameters is much smaller than the transverse. We assume that the thermophysical parameters are constant and that the equation of the Archimedes spiral in polar coordinates is $r = A\theta$, where $A > 0$. Under these assumptions the hydrodynamics and energy equations take the form

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=F_x-\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu\frac{\partial^2 u}{\partial y^2},$$

$$-\frac{u^2}{R(x)}=F_y-\frac{1}{\rho}\frac{\partial p}{\partial y},\quad \frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=a\frac{\partial^2 T}{\partial y^2}, \tag{2}$$

Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 33, No. 5, pp. 807-815, November, 1977. Original article submitted October 8, 1976.

where $R(x) = A(\theta^2+1)^{3/2}/(\theta^2+2)$; and F_x, F_y are the components of the body forces along the coordinate axes.

The body forces acting on a unit mass of the liquid film are the centrifugal force $F_{ce} = \omega^2 R(x)$ and the Coriolis force $F_{co} = 2\bar{\omega} \times \bar{v}$. The components of the body forces along the coordinate axes have the form

$$F_x = \omega^2 R(x)\cos\alpha \pm 2\omega v, \quad F_y = -\omega^2 R(x)\sin\alpha \mp 2\omega u, \tag{3}$$

where the upper signs correspond to a counterclockwise rotation of the spiral and the lower to clockwise rotation. The angle α is related to the polar angle θ by the equations

$$\sin\alpha = \theta/\sqrt{\theta^2+1}; \quad \cos\alpha = 1/\sqrt{\theta^2+1}.$$

The boundary conditions for Eqs. (1) and (2) are

$$\begin{aligned} &y = 0 \quad u = 0, \quad v = 0, \quad T = T_w\,; \\ &y = H(x) \quad \frac{\partial u}{\partial y} = 0, \quad p = \text{const}, \quad T = T_f, \end{aligned} \tag{4}$$

where the equation of the surface H(x) is determined from the solution of Eqs. (1) and (2), taking account of the kinematic condition $v_N = u_N \dfrac{dH}{dx}$ on the boundary surface. Introducing the dimensionless variables

$$u = u_p\bar{u}, \quad x = \delta_p \mathrm{Re}\,\bar{x}, \quad y = \delta_p\bar{y}, \quad T = \frac{T - T_w}{T_f - T},$$

where $\delta_p = \sqrt[3]{\mathrm{Re}/\mathrm{Ga}}$, obtained from the solution of Eqs. (1) and (2) in the stabilization region in the variables (θ, y), where $dx = A\sqrt{\theta^2+1}\,d\theta$, Eqs. (1) and (2) and boundary conditions (4) take the form (omitting bars over symbols)

$$\begin{aligned} &\frac{E5E1\,\mathrm{Re}}{\sqrt{\theta^2+1}}\,u\,\frac{\partial u}{\partial\theta} + v\,\frac{\partial u}{\partial y} = F_x - \frac{E5E1\,\mathrm{Re}}{\sqrt{\theta^2+1}}\,\frac{\partial p}{\partial\theta} + 3\,\frac{\partial^2 u}{\partial y^2}, \\ &-E5E1\,\frac{u^2}{R(x)} = F_y - \frac{\partial p}{\partial y}, \\ &\frac{E5E1\,\mathrm{Re}}{\sqrt{\theta^2+1}}\cdot\frac{\partial u}{\partial\theta} + \frac{\partial v}{\partial y} = 0, \end{aligned} \tag{5}$$

$$\frac{E5E1\,\mathrm{Re}}{\sqrt{\theta^2+1}}\,u\,\frac{\partial T}{\partial\theta} + v\,\frac{\partial T}{\partial y} = \frac{3}{\mathrm{Pr}}\,\frac{\partial^2 T}{\partial y^2} \tag{6}$$

$$\begin{aligned} &\text{at } x = 0 \quad T = 0, \\ &\text{at } y = 0 \quad u = v = 0, \quad T = 0, \\ &\text{at } y = \frac{H(x)}{\delta_p} \quad \frac{\partial u}{\partial y} = 0, \quad \frac{\partial p}{\partial\theta} = 0, \quad T = 1, \end{aligned} \tag{7}$$

where

$$\begin{aligned} &F_x = 9\,\frac{\theta^2+1}{\theta^2+2} \pm \frac{6\mathrm{Ga}^{1/2}E5^{1/2}E1^2}{\mathrm{Re}}\,v; \\ &F_y = -\frac{9}{\mathrm{Re}}\,\theta\,\frac{\theta^2+1}{\theta^2+2} \mp \frac{6\mathrm{Ga}^{1/2}E5^{1/2}E1^2}{\mathrm{Re}}\,u. \end{aligned} \tag{8}$$

We solve the problem by the method of equal flow-rate surfaces, which is related to the group of collocation methods [2]. We introduce the lines $y_k = y_k(x)$ into the flow field, and the notation

$$u_k(x) = u[x, y_k(x)], \quad v_k(x) = v[x, y_k(x)], \quad T_k(x) = T[x, y_k(x)].$$

Then v_k and u_k are connected by the relation

$$v_k(x) = \frac{E5E1\,\mathrm{Re}}{\sqrt{\theta^2+1}}\,u_k(x)\,\frac{dy_k}{d\theta}, \tag{9}$$

which follows from the conservation of flow-rate condition and the equation of continuity. In addition, the equation of continuity is equivalent to the following system of equations:

$$\int\limits_{y_{k-1}(x)}^{y_k(x)} u\,dy = \text{const}_k \quad k = 2, 3, \ldots, N. \tag{10}$$

Evaluating these integrals by the trapezoidal rule with a uniform error estimate with respect to the variable θ having the third order of smallness, we obtain a system of nonlinear algebraic equations of the form

$$(y_k(x) - y_{k-1}(x))\,(u_k(x) + u_{k-1}(x)) = \mathrm{const}_k + O_3. \tag{11}$$

By differentiating both sides of (11) with respect to θ we obtain a system of ordinary differential equations for determining the surfaces of equal flow rate $y_k(x)$

$$\frac{dy_k}{d\theta} = \frac{dy_{k-1}}{d\theta} - \frac{y_k - y_{k-1}}{u_k + u_{k-1}} \left(\frac{du_k}{d\theta} + \frac{du_{k-1}}{d\theta} \right). \tag{12}$$

The derivatives with respect to the independent variable θ have the form

$$\frac{d\varphi_k}{d\theta} = \left[\frac{\partial \varphi_k}{\partial \theta} + \frac{\partial \varphi_k}{\partial y_k} \frac{dy}{d\theta} \right]_{y=y_k}, \quad \varphi_k = u_k,\ T_k. \tag{13}$$

Substituting $\frac{\partial \varphi_k}{\partial \theta}$ from (13) into (5) and (6) and using (9) we obtain

$$\frac{E5E1\,\mathrm{Re}}{\sqrt{\theta^2+1}}\, u_k \frac{du_k}{d\theta} = F_{xk} - \frac{E5E1\,\mathrm{Re}}{\sqrt{\theta^2+1}} \left[\frac{dp_k}{d\theta} - \frac{\partial p_k}{\partial y_k} \frac{dy_k}{d\theta} \right] + 3 \frac{\partial^2 u_k}{\partial y_k^2}, \tag{14}$$

$$\frac{dp_k}{d\theta} = \frac{dp_{k-1}}{d\theta} + \frac{dM_k}{d\theta}, \tag{15}$$

$$\frac{E5E1\,\mathrm{Re}}{\sqrt{\theta^2+1}}\, u_k \frac{dT_k}{d\theta} = \frac{3}{\mathrm{Pr}} \frac{\partial^2 T_k}{\partial y_k^2}, \tag{16}$$

where

$$M_k(\theta) = \int_{y_{k-1}}^{y_k} \left(E5E1 \frac{u^2}{R(\theta)} + F_y \right) dy;$$

$$\frac{\partial p_k}{\partial y_k} = F_{yk} + E5E1 \frac{u_k^2}{R(\theta)}, \quad k = 2, 3, \ldots, N.$$

To evaluate the second derivatives with respect to y in Eqs. (14) and (15) we write the solution for φ_k as an expansion in a complete set of functions

$$\varphi_k(x) = \sum_{j=1}^{N} B_j(x) F_{kj}(x), \quad j = 1, 2, \ldots, N, \quad k = 1, 2, \ldots, N, \tag{17}$$

where

$$\varphi_k(x) = u_k,\ T_k; \quad B_j(x) = A_j,\ A1_j; \quad F_{kj}(x) = V_{kj},\ K1_{kj}.$$

In the present paper the systems of basis functions for velocity and temperature were chosen, respectively, in the form

$$V_{kj}(x) = \left(\frac{j+1}{j} - \eta_k \right) \eta_k^j, \tag{18}$$

$$K1_{kj}(x) = \frac{1}{2} (\eta_k^j - \eta_k^{j+1}) + \eta_k^{j+1}. \tag{19}$$

The orthogonal Tschebyscheff polynomials of the first kind were used also. For the velocity and temperature these had the form

$$V_{kj}(x) = T_{j+1}(\eta_k) - T_{j+1}(0) - [T_j(\eta_k) - T_j(0)] \left(\frac{j+1}{j} \right)^2, \tag{20}$$

$$K1_{kj}(x) = T_{j+1}(\eta_k) + T_{j+1}(0)(2\eta_k^2 - \eta_k - 1), \tag{21}$$

$$\eta_k(x) = y_k(x)/H(x), \quad j = 1, 2, \ldots, N.$$

The results obtained by using different complete sets of functions did not differ appreciably, but the required accuracy was attained with fewer terms when using the Tschebyscheff polynomials.

We require that the values of the velocity (temperature) determined by Eq. (17) agree with u_k (x) T_k (x) on the lines $y_k(x)$. Then we obtain a system of linear algebraic equations for the coefficients $B_j(x)$

$$\varphi_k(x)=\sum_{j=1}^{N} B_j(x)F_{kj}(x). \tag{22}$$

Determining the $B_j(x)$ from (22) we find from (17) the derivatives of the velocity and temperature with respect to y in Eqs. (14) and (16).

The system of nonlinear ordinary differential equations (12), (14)-(16) was solved by the Runge–Kutta method. Since the pressure gradient in the first layer $dp_1/d\theta$ is unknown, the right-hand sides of Eqs. (12), (14)-(16) were determined in two stages: 1) the pivotal coefficients were calculated and used to find $dp_1/d\theta$, and 2) the right-hand sides were evaluated correctly. With this in mind we reduce Eqs. (12), (14)-(16) to the form

$$\begin{gathered}\frac{du_k}{d\theta}+L_k\frac{dp_k}{d\theta}+N_k\frac{dy_k}{d\theta}=R_k,\\ \frac{dy_k}{d\theta}-\frac{dy_{k-1}}{d\theta}+S_k\frac{du_k}{d\theta}+S_k\frac{du_{k-1}}{d\theta}=0,\\ \frac{dp_k}{d\theta}-\frac{dp_{k-1}}{d\theta}+Q_k\frac{du_k}{d\theta}+T_k\frac{du_{k-1}}{d\theta}=\Omega_k,\ k=2,\ 3,\ \ldots,\ N,\end{gathered} \tag{23}$$

where

$$L_k=\frac{1}{u_k}\ ;\ N_k=L_k\left(\frac{9}{\mathrm{Re}}\,\theta\,\frac{\theta^2+1}{\theta^2+2}-\frac{E5E1\,(\theta^2+2)}{(\theta^2+1)^{3/2}}\,u_k^2\right);$$

$$R_k=\frac{3L_k}{E5E1\,\mathrm{Re}}\left[\sqrt{\theta^2+1}\,\frac{\partial^2 u_k}{\partial y_k^2}+3\,\frac{(\theta^2+1)^{3/2}}{\theta^2+2}\right];\ S_k=\frac{y_k-y_{k-1}}{u_k+u_{k-1}};$$

$$\begin{aligned}Q_k=S_k\Bigg[&(u_k^2+u_{k-1}^2)\frac{E5E1\,(\theta^2+2)}{2\,(\theta^2+1)^{3/2}}-\frac{9}{\mathrm{Re}}\,\theta\,\frac{\theta^2+1}{\theta^2+2}\mp\\ &\mp\frac{3\mathrm{Ga}^{1/2}E5^{1/2}E1^2}{\mathrm{Re}}(u_k+u_{k-1})\Bigg]-(y_k-y_{k-1})\times\\ &\times\left[u_k\frac{E5E1\,(\theta^2+2)}{(\theta^2+1)^{3/2}}\mp\frac{3\mathrm{Ga}^{1/2}E5^{1/2}E1^2}{\mathrm{Re}}\right];\end{aligned}$$

$$T_k=Q_k+(y_k-y_{k-1})(u_k-u_{k-1})\frac{E5E1\,(\theta^2+2)}{(\theta^2+1)^{3/2}};$$

$$\Omega_k=(y_{k-1}-y_k)\left[(u_k^2+u_{k-1}^2)\,\frac{E5E1\theta\,(\theta^2+4)}{2\,(\theta^2+1)^{5/2}}+\frac{9}{\mathrm{Re}}\,\frac{\theta^4+5\theta^2+2}{(\theta^2+2)^2}\right].$$

We write the functions being sought $\frac{du_k}{d\theta}$, $\frac{dy_k}{d\theta}$, $\frac{dp_k}{d\theta}$ in the form of pivotal relations

$$\frac{d\psi_k}{d\theta}=\chi_k+\hat{\chi}_k\frac{dP_1}{d\theta}, \tag{24}$$

where

$$\psi_k=u_k,\ p_k,\ y_k;\ \chi_k=U_k,\ P_k,\ Y_k;\ \hat{\chi}_k=\hat{U}_k,\ \hat{P}_k,\ \hat{Y}_k.$$

Substituting Eqs. (24) into (23) and taking account of the fact that χ_k and $\hat{\chi}_k$ are known, we obtain explicit expressions for the pivotal coefficients in the form of the following recurrence relations:

$$\begin{gathered}U_k=[R_k-N_k\,(Y_{k-1}-S_kU_{k-1})-L_k\,(\Omega_k+P_{k-1}-T_kU_{k-1})]\,(1-L_kQ_k-N_kS_k)^{-1},\\ \hat{U}_k=[N_k(S_k\hat{U}_{k-1}-\hat{Y}_{k-1})+L_k\,(T_k\hat{U}_{k-1}-\hat{P}_{k-1})]\,(1-L_kQ_k-N_kS_k)^{-1},\\ Y_k=-S_kU_{k-1}+Y_{k-1}-S_kU_k,\ \hat{Y}_k=-S_k\hat{U}_{k-1}+\hat{Y}_{k-1}-S_k\hat{U}_k,\\ P_k=\Omega_k+P_{k-1}-T_kU_{k-1}-Q_kU_k,\ \hat{P}_k=\hat{P}_{k-1}-T_k\hat{U}_{k-1}-\\ -Q_k\hat{U}_k,\ k=2,\ 3,\ \ldots,\ N.\end{gathered} \tag{25}$$

From the given boundary conditions

$$\frac{du_1}{d\theta}=\frac{dy_1}{d\theta}=\frac{dp_N}{d\theta}=0 \tag{26}$$

and from Eqs. (23) we find the values of the pivotal coefficients for k = 2. Then, using Eqs. (25), we find the values of the pivotal coefficients for any k. In particular, for k = N

$$\frac{dp_N}{d\theta}=p_N+\hat{p}_N\frac{dp_1}{d\theta},$$

from which

$$\frac{dp_1}{d\theta}=\left(\frac{dp_N}{d\theta}-p_N\right)/\hat{p}_N. \tag{27}$$

Then by a reverse pivotal we calculate the values of the right-hand sides of the system of differential equations (24).

After determining the velocity field u_k, the system of differential equations was solved for the temperature in the liquid layer by the Runge–Kutta method. We find the thermal flux on the wall of the spiral apparatus from the equation

$$\beta(x)=-a\left(\frac{\partial T}{\partial y}\right)_{y=0}=\frac{d}{dx}\int_0^{y_N}uTdy, \tag{28}$$

which is obtained after integrating the energy equation across a liquid film of variable thickness, using Eq. (9) at the boundary. After averaging the flux over some characteristic values L we obtain*

$$\beta_T=\frac{1}{L}\int_0^L\beta dx=\frac{1}{L}\left(\int_0^{y_N}uTdy\right)_{x_1=L}\cdot\frac{u_p\delta_p}{L}\left(\int_0^{\bar{y}_N}\bar{u}Tdy\right)_{\bar{x}_1=\frac{L}{\delta_p\,\mathrm{RePr}}}=\frac{u_p\delta_p}{L}HT\Big|_{\bar{x}_1=\frac{L}{\delta_p\,\mathrm{RePr}}}. \tag{29}$$

Using the algorithm described above, the velocity field, the temperature, and the surface of separation were calculated in the entrance region as functions of the width of the slit, the Reynolds number Re, and the dimensionless characteristic of the spiral E5.

Figure 1 shows the characteristic form of the development of the velocity profile in the liquid film, and Fig. 2 the dimensionless liquid-film thickness as a function of the dimensionless length of the spiral. These figures show that the width of the slit has an appreciable effect on the accelerated flow of the liquid film up to $h_0/\delta_p = 7$. For $h_0/\delta_p > 7$ the slit width has a negligible effect on the accelerated flow of the liquid film.

Figure 3 shows the surface velocity as a function of the dimensionless length of the spiral. The development of the surface velocity follows the same rules as the development of the liquid-film velocity [3]. Figure 4 gives the characteristic form of HT^2 as a function of the dimensionless length of the spiral for various values of the hydrodynamic parameters. These figures show that larger values of E1, E5, Re and Pr correspond to larger values of HT^2.

The value of HT^2 is approximated within 10% in the range of parameters investigated by the expression

$$HT^2=(3.9+0.75E1+0.9E5+0.001\,\mathrm{Re}+0.007\,\mathrm{Pr})\bar{x}+0.005E1. \tag{30}$$

The expression for the average coefficient of heat transfer from the liquid film to the wall of the spiral heat-transfer apparatus, taking account of (30), has the form

$$\beta_T=\frac{u_p^{1/2}a^{1/2}}{L^{1/2}3^{1/2}}\sqrt{3.9+0.75E1+0.9E5+0.001\mathrm{Re}+0.007\mathrm{Pr}+0.005\frac{E1\delta_p\mathrm{RePr}}{L}}. \tag{31}$$

Table 1 compares the results calculated by the two methods for E1 = 1, E5 = 0.5, Pr = 10, h_0 = 0.3 cm, L = 100 cm, and $\nu = 10^{-2}$ cm^2/sec; single and double primes denote, respectively, values for the integral-relations method and for the method presented, where

*In certain cases it is convenient to solve the energy equation by using the dimensionless longitudinal coordinate in the form $x = \delta_p\mathrm{RePr}\bar{x}$. Then the small parameter in the energy equation disappears for the higher derivative.

TABLE 1

Re	x'_k/h_0	x''_k/δ	β''_{av}/β'_{av}
50	25,25	14,75	1,05
100	30,5	20,5	1,128
300	51,5	43,5	1,303
500	72,5	66,5	1,439
1000	125	124	1,731

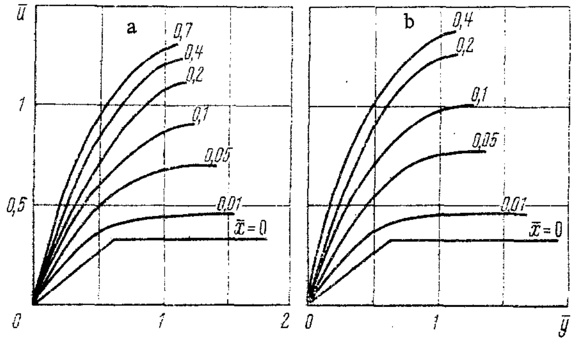

Fig. 1. Dimensionless velocity as a function of the transverse coordinate for E5 = 0.1 and Hl = 3; a) Re = 100; b) Re = 500.

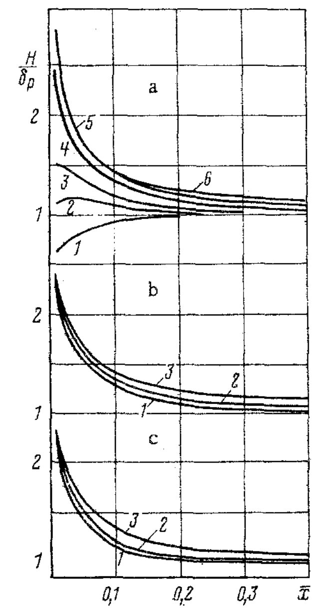

Fig. 2

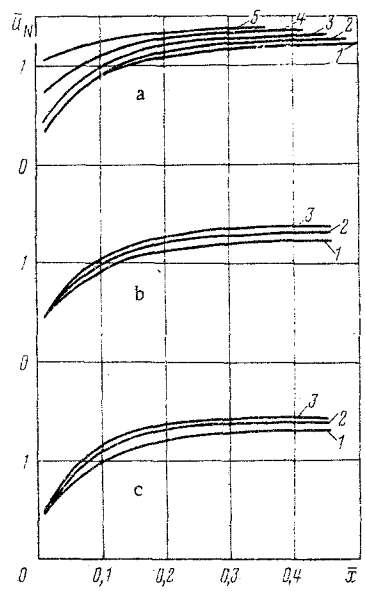

Fig. 3

Fig. 2. Dimensionless thickness of liquid film as a function of dimensionless length of spiral: a) Re = 300; E5 = 0.1; 1) H1 = 0.5; 2) 1; 3) 1.5; 4) 3; 5) 7; 6) 10; b) E5 = 0.1; H1 = 3; 1) Re = 500; 2) 300; 3) 100; c) Re = 300; H1 = 3; 1) E5 = 1; 2) 0.5; 3) 0.1.

Fig. 3. Dimensionless surface velocity as a function of dimensionless length of spiral: a) Re = 300; E5 = 0.1; 1) H1 = 10; 2) 7; 3) 3; 4) 1.5; 5) 1; b) E5 = 0.1; H1 = 3; 1) Re = 100; 2) 300; 3) 500; c) Re = 300; H1 = 3; 1) E5 = 0.1; 2) 0.5; 3) 1.

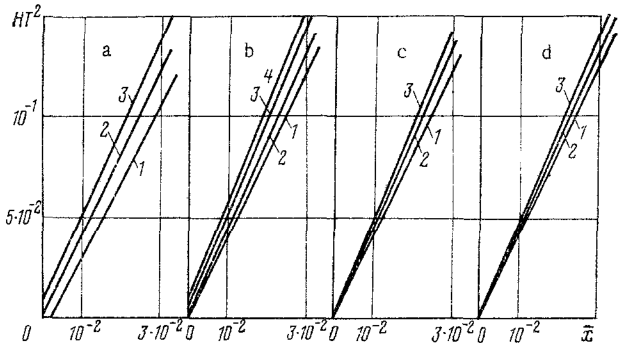

Fig. 4. HT^2 as a function of dimensionless length of spiral: a) Re = 100; E5 = 0.1; E1 = 0.1; 1) Pr = 10; 2) 30; 3) 100; b) Re = 100; E5 = 0.1; Pr = 30; 1) E1 = 0.1; 2) 0.4; 3) 1; 4) 1.6; c) Re = 100; E1 = 0.1; Pr = 30; 1) E5 = 0.1; 2) 0.5; 3) 1; d) E5 = 0.1; E1 = 0.1; Pr = 30; 1) Re = 100; 2) 500; 3) 1000.

$$x'_k = [(0.08E1 + 0.025)\,\mathrm{Re} + 15E1 + 5]\,h_0;\ x''_k = \left[(0.09E5 + 0.07)\,\mathrm{Re} + \frac{4.5}{E1E5}\right]\delta;$$

$$\beta_{av} = \beta_{in}\frac{x_k}{L} + \beta_w\frac{L - x_k}{L}.$$

As can be seen from the table, there is a difference in the calculated values of β. This difference can be accounted for in the following way.

In the integral-relations method [1] the basis functions were second-order polynomials, while in the proposed method they were N-th-order (N = 10) orthogonal polynomials. These N-th-order polynomials can more accurately trace the complex variations of all the hydrodynamic parameters (velocity, film thickness) in the entrance region. In the integral-relations method a parabolic velocity profile is specified a priori, i.e., the profile which occurs in the stabilization region, and the termination of the calculation was determined solely by the degree of approximation of the film thickness. In the proposed method the termination of the calculation, and consequently also the entrance length x_k, depend on the degree of approximation of the values of both the surface velocity and the film thickness in the stabilization region. This caused the difference in the calculated values of x_k (Table 1).

Since the dimensionless size of the active region $\bar{x} = x/\delta\mathrm{Re}$ decreases with increasing Re (Fig. 1), i.e., the velocity profile is shaped in the smaller dimensionless part, further difficulties arise in describing the deformation of the velocity-profile by the integral-relations method, and consequently the difference in the results calculated by the two methods is increased. This is actually the case (Table 1). Moreover, the calculated values agree up to Re = 100.

NOTATION

a, thermal diffusivity; ν, viscosity; ρ, density; h_0, initial thickness of liquid film; A, characteristic of spiral; R(x), radius of curvature of spiral; α, angle between positive direction of tangent to spiral and radius vector to point under consideration, calculated from the expression $\tan\alpha = r(\theta)/r'(\theta)$; T_W, T_f, temperatures at wall of spiral and at film surface, respectively, H(x), equation of surface determined from solution of problem; p(x, y), hydrostatic pressure; $x = A(\theta\sqrt{\theta^2+1} + \ln|\theta + \sqrt{\theta^2+1}|/2$, running length of spiral; $E5 = h_0/A$, its dimensionless characteristic; $\mathrm{Re} = 3q/\nu$, modified Reynolds number; $\mathrm{Ga} = \omega^2 A h_0^3/\nu^2$, Galileo number, $E1 = \sqrt[3]{\mathrm{Re}/\mathrm{Ga}} = \delta/h_0$, ratio of thickness of boundary layer to initial thickness; $H1 = h_0/\delta$, ratio of initial thickness of liquid film to running thickness; x_k, entrance length.

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